06

We have already deal with equations in one variable and two variables. We have also solved some word problems by translating them in the form of equation. But it is not necessary that every word problems translate into equation. It may involve the sign(s) '$<$', '$>$', '$\leq$' and '$\geq$'.

In this chapter, we will study about inequalities, linear inequalities in one and two variables and various application involving these inequalities.

# LINEAR INEQUALITIES

**CHAPTER CHECKLIST**

- Inequality and Linear Inequalities in One Variable
- System of Inequalities in One Variable and Their Solutions
- Applications of Linear Inequalities in One Variable

## |TOPIC 1|
## Inequality and Linear Inequalities in One Variable

### INEQUALITY

Two real numbers or two algebraic expressions related by the symbols $>, <, \leq$ or $\geq$ form an inequality or inequation.

In other words, a statement involving the symbols $>, <, \leq$ or $\geq$ is called an **inequality** or **inequation**.

Here, the symbols $<$ (less than), $>$ (greater than), $\leq$ (less than or equal to) and $\geq$ (greater than or equal to) are known as symbol of **inequalities**.

e.g. $5 < 7, x \leq 2, x + y \geq 11$

### Types of Inequalities

(i) **Numerical inequality** An inequality which does not involve any variable is called a numerical **inequality**.

e.g. $4 > 2,\ 8 < 21$

(ii) **Literal inequality** An inequality which have variables is called literal inequality.

e.g. $x < 7,\ y \geq 11,\ x - y \leq 4$

(iii) **Strict inequality** An inequality which have only $<$ or $>$ is called strict inequality.

e.g. $3x + y < 0,\ x > 7$

(iv) **Slack inequality** An inequality which have only $\geq$ or $\leq$ is called slack inequality.

e.g. $3x + 2y \leq 0,\ y \geq 4$

# LINEAR INEQUALITY

An inequality is said to be linear, if the variable (s) occurs in first degree only and there is no term involving the product of the variables. e.g. $ax + b \le 0$, $ax + by + c > 0$, $ax \le 4$.

## Linear Inequality in One Variable

A linear inequality which has only one variable, is called linear inequality in one variable.

e.g. $ax + b < 0$, where $a \ne 0$

**Note**

An inequality in one variable, in which degree of variable is 2, is called quadratic inequality in one variable.

e.g. $ax^2 + bx + c \ge 0$, $a \ne 0$; $3x^2 + 2x + 1 \le 0$

## Linear Inequality in Two Variables

A linear inequality which have only two variables, is called linear inequality in two variables.

e.g. $3x + 11y \le 0$, $4t + 3y > 0$

### CONCEPT OF INTERVALS ON A NUMBER LINE

On number line or real line, various types of infinite subsets, known as intervals, are defined below

#### CLOSED INTERVAL

If $a$ and $b$ are real numbers, such that $a < b$, then the set of all real numbers $x$, such that $a \le x \le b$, is called a closed interval and is denoted by $[a, b]$.

$\therefore$ $[a, b] = \{x : a \le x \le b, x \in R\}$

*On the number line, $[a, b]$ may be represented as follows*


Here, end points of the interval i.e. $a$ and $b$ are included in the interval. So, on number line, draw filled circle (•) at $a$ and $b$.

#### OPEN INTERVAL

If $a$ and $b$ are real numbers, such that $a < b$, then the set of all real numbers $x$, such that $a < x < b$, is called an open interval and is denoted by $(a, b)$ or $]a, b[$.

$\therefore$ $(a, b) = \{x : a < x < b, x \in R\}$

*On the number line, $(a, b)$ may be represented as follows*


Here, end points of the interval i.e. $a$ and $b$ are not included in the interval. So, on number line, draw open circle (o) at $a$ and $b$.

#### SEMI-OPEN OR SEMI-CLOSED INTERVALS

If $a$ and $b$ are real numbers, such that $a < b$.

Then, $(a, b] = \{x : a < x \le b, x \in R\}$

and $[a, b) = \{x : a \le x < b, x \in R\}$

are known as semi-open or semi-closed intervals.

*On the number line, these intervals may be represented as follows*



# SOLUTION OF AN INEQUALITY

Any solution of an inequality is the value(s) of variable(s) which makes it a true statement.

We can find the solutions of an inequality by hit and trial method but it is not very efficient because this method is time consuming and sometimes not feasible. So, we solve inequalities with systematic technique.

*Some properties or rules which are used to solve the inequalities, are given below*

## Addition or Subtraction

Some number may be added (or subtracted) to (from) both sides of an inequality i.e. if $a > b$, then for any number $c$,

$$a + c > b + c \text{ or } a - c > b - c$$

e.g. (i) $10 > 5 \Rightarrow 10 + 7 > 5 + 7$ [adding 7 on both sides]

$\Rightarrow 17 > 12$, which is true.

(ii) $-8 > -13 \Rightarrow -8 - 2 > -13 - 2$

[subtracting 2 from both sides]

$\Rightarrow -10 > -15$, which is true.

## Multiplication or Division

If both sides of an inequality are multiplied (or divided) by the same positive number, then the sign of inequality remains the same. But when both sides are multiplied (or divided) by the same negative number, then the sign of inequality is reversed.

Let $a, b$ and $c$ be three real numbers, such that $a > b$ and $c \ne 0$.

(i) If $c > 0$, then $\frac{a}{c} > \frac{b}{c}$ and $ac > bc$.

(ii) If $a > b$ and $c < 0$, then $\frac{a}{c} < \frac{b}{c}$ and $ac < bc$.

e.g. (a) $6 > 4 \Rightarrow 6 \times 2 > 4 \times 2$
[multiplying both sides by 2]

$\Rightarrow \quad 12 > 8$, which is true.

(b) $-32 < -24 \Rightarrow \frac{-32}{-8} > \frac{-24}{-8}$
[dividing both sides by $-8$]

$\Rightarrow 4 > 3$, which is true.

## SOLUTION SET

The set of all solutions of an inequality is called the solution set of the inequality.

### Algebraic Solution of Linear Inequalities in One Variable

Any solution of an linear inequality in one variable is a value of the variable which makes it a true statement.

e.g. $x = 1$ is the solution of the linear inequality $4x + 7 > 0$.

#### METHOD TO SOLVE A LINEAR INEQUALITY IN ONE VARIABLE

*Step* I Collect all terms involving the variable ($x$) on one side and constant terms on other side with the help of above rules and then reduce it in the form $ax < b$ or $ax \le b$ or $ax > b$ or $ax \ge b$.

*Step* II Divide this inequality by the coefficient of variable ($x$). This gives the solution set of given inequality.

*Step* III Write the solution set.

**EXAMPLE |1|** Solve $24x < 100$, when

(i) $x$ is a natural number. (ii) $x$ is an integer. **[NCERT]**

Given inequality is of the form $ax < b$, so divide both sides by 24.

***Sol.*** We have, $24x < 100$

On dividing both sides by 24, we get

$$= \frac{24x}{24} < \frac{100}{24} \Rightarrow x < \frac{25}{6}$$

(i) When $x$ is a natural number, then solutions of the inequality $x < \frac{25}{6}$, are all natural numbers, which are less than $\frac{25}{6}$. In this case, the following values of $x$ make the statement true.

$$x = 1, 2, 3, 4$$

Hence, the solution set of inequality is {1, 2, 3, 4}.

(ii) When $x$ is an integer.

In this case, solutions of given inequality are

$$\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4$$

Hence, the solution set of inequality is

$$\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

**EXAMPLE |2|** Solve the linear inequality $-5x + 25 > 0$.

***Sol.*** We have, $-5x + 25 > 0$

On adding $5x$ to both sides, we get

$$-5x + 25 + 5x > 0 + 5x$$

$$\Rightarrow \quad 25 > 5x \Rightarrow 5x < 25$$

On dividing both sides by 5, we get

$$\Rightarrow \quad \frac{5x}{5} < \frac{25}{5} \Rightarrow x < 5$$

Hence, the required solution set is $(-\infty, 5)$.

**EXAMPLE |3|** Solve the linear inequality $3x - 5 < x + 7$, when **[NCERT Exemplar]**

(i) $x$ is a natural number. (ii) $x$ is a whole number.
(iii) $x$ is an integer. (iv) $x$ is a real number.

***Sol.*** We have, $3x - 5 < x + 7$

$\Rightarrow \quad 3x - 5 + 5 < x + 7 + 5$ [adding 5 on both sides]

$\Rightarrow \quad 3x < x + 12$

$\Rightarrow \quad 3x - x < x + 12 - x$
[subtracting $x$ from both sides]

$\Rightarrow \quad 2x < 12$

$\Rightarrow \quad \frac{2x}{2} < \frac{12}{2}$ [dividing both sides by 2]

$\Rightarrow \quad x < 6$

Now, if

(i) $x$ is natural number, then the solution set is {1, 2, 3, 4, 5}.

(ii) $x$ is a whole number, then the solution set {0, 1, 2, 3, 4, 5}.

(iii) $x$ is an integer, then the solution set is

$$\{\ldots, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$$

(iv) $x$ is a real number, then the solution set is $\{x : x \in R \text{ and } x < 6\}$, i.e. any real number less than 6. This can also be written as $(-\infty, 6)$.

**EXAMPLE |4|** Solve the inequality $\frac{5 - 3x}{3} \le \frac{x}{6} - 5$.

***Sol.*** We have, $\frac{5 - 3x}{3} \le \frac{x}{6} - 5$

$\Rightarrow \quad \frac{5 - 3x}{3} \le \frac{x - 30}{6}$

$\Rightarrow \quad \frac{5 - 3x}{3} \times 6 \le \frac{x - 30}{6} \times 6$
[multiplying both sides by 6]

$\Rightarrow \quad 2(5 - 3x) \le x - 30$

$\Rightarrow \quad 10 - 6x \le x - 30$

$\Rightarrow 10 - 6x - 10 \le x - 30 - 10$
[subtracting 10 from both sides]

$\Rightarrow \quad -6x \le x - 40$

$\Rightarrow \quad -6x - x \le x - 40 - x$
[subtracting $x$ from both sides]

$\Rightarrow \quad -7x \le -40$

$\Rightarrow \quad 7x \ge 40$ [multiplying by $-1$ both sides]

$\Rightarrow \quad \frac{7x}{7} \ge \frac{40}{7}$ [dividing by 7 both sides]

$\Rightarrow \quad x \ge \frac{40}{7}$

i.e. $\quad x \in \left[\frac{40}{7}, \infty\right)$

Hence, the required solution set is $\left[\frac{40}{7}, \infty\right)$.

## REPRESENTATION OF SOLUTION OF LINEAR INEQUALITY IN ONE VARIABLE ON NUMBER LINE

To represent the solution of a linear inequality in one variable on a number line, *use the following rules*

(i) To represent $x < a$ (or $x > a$) on a number line, put a circle (o) on the number $a$ and dark the line to the left (or right) of the number $a$.

(ii) To represent $x \le a$ (or $x \ge a$) on a number line, put a dark circle (•) on the number $a$ and dark the line to the left (or right) of the number $a$.

**EXAMPLE |5|** Solve the following inequality and show the graph of the solution in each case on number line.

(i) $\frac{x}{4} < \frac{5x-2}{3} - \frac{7x-3}{5}$ (ii) $x + \frac{x}{2} + \frac{x}{3} < 11$

(iii) $\frac{1}{2}\left(\frac{3}{5}x + 4\right) \ge \frac{1}{3}(x-6)$ (iv) $\frac{(2x-1)}{3} \le \frac{(3x-2)}{4} - \frac{(2-x)}{5}$

***Sol.*** (i) We have, $\frac{x}{4} < \frac{5x-2}{3} - \frac{7x-3}{5}$

$\Rightarrow \quad \frac{x}{4} < \frac{5(5x-2)-3(7x-3)}{15}$

$\Rightarrow \quad \frac{x}{4} < \frac{25x-10-21x+9}{15}$

$\Rightarrow \quad 60\left(\frac{x}{4}\right) < \frac{(4x-1)}{15} \times 60$

[multiplying both sides by LCM $(4, 15) = 60$]

$\Rightarrow \quad 15x < 4(4x-1)$

$\Rightarrow \quad 15x < 16x - 4$

$\Rightarrow \quad 15x + 4 < 16x - 4 + 4$ [adding 4 on both sides]

$\Rightarrow \quad 15x + 4 < 16x$

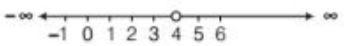
$\Rightarrow \quad 15x + 4 - 15x < 16x - 15x$

[subtracting $15x$ from both sides]

$\Rightarrow \quad 4 < x$ or $x > 4$

$\therefore \quad x \in (4, \infty)$

On number line, it can be represented as




Here, the dark portion on the number line represents the solution of inequality.

(ii) We have, $\quad x + \frac{x}{2} + \frac{x}{3} < 11$

$\Rightarrow \quad \frac{6x+3x+2x}{6} < 11 \Rightarrow \frac{11x}{6} < 11$

On multiplying both sides by $\frac{6}{11}$, we get

$\frac{11x}{6} \times \frac{6}{11} < 11 \times \frac{6}{11} \Rightarrow x < 6$

$\therefore \quad x \in (-\infty, 6)$

This can be represented on number line as


Here, the dark portion on the number line represents the solution of inequality.

(iii) We have, $\frac{1}{2}\left(\frac{3}{5}x + 4\right) \ge \frac{1}{3}(x-6)$

$\Rightarrow \quad \frac{1}{2}\left(\frac{3x+20}{5}\right) \ge \frac{1}{3}(x-6)$

$\Rightarrow \quad \frac{1}{10}(3x+20) \ge \frac{1}{3}(x-6)$

$\Rightarrow \quad \frac{30}{10}(3x+20) \ge \frac{30}{3}(x-6)$

[multiplying both sides by LCM $(10, 3) = 30$]

$\Rightarrow \quad 3(3x+20) \ge 10(x-6)$

$\Rightarrow \quad 9x + 60 \ge 10x - 60$

$\Rightarrow \quad 9x + 60 - 60 \ge 10x - 60 - 60$

[subtracting 60 from both sides]

$\Rightarrow \quad 9x \ge 10x - 120$

$\Rightarrow \quad 9x - 10x \ge 10x - 120 - 10x$

[subtracting $10x$ from both sides]

$\Rightarrow \quad -x \ge -120 \Rightarrow x \le 120$

[multiplying both sides by $-1$]

$\therefore \quad x \in (-\infty, 120]$

On number line, it can be represented as


Here, the dark portion on the number line represents the solution of inequality.

(iv) We have, $\frac{2x-1}{3} \le \frac{3x-2}{4} - \frac{2-x}{5}$

On multiplying both sides by LCM of 3, 4 and 5 i.e. 60, we get

$\frac{2x-1}{3} \times 60 \le \frac{3x-2}{4} \times 60 - \frac{2-x}{5} \times 60$

$\Rightarrow \quad 20(2x-1) \le 15(3x-2) - 12(2-x)$

$\Rightarrow \quad 40x - 20 \le 45x - 30 - 24 + 12x$

$\Rightarrow \quad 40x - 20 \le 57x - 54$

$\Rightarrow \quad 40x - 20 + 20 \le 57x - 54 + 20$

[adding 20 both sides]

$\Rightarrow \quad 40x \le 57x - 34$

$\Rightarrow \quad 40x - 57x \le 57x - 34 - 57x$

[subtracting $57x$ from both sides]

$\Rightarrow \quad -17x \le -34$

$\Rightarrow \quad \frac{-17x}{17} \le \frac{-34}{17}$ [dividing both sides by 17]

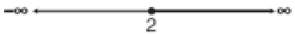
$\Rightarrow \quad -x \le -2$

$\Rightarrow \quad x \ge 2$

[multiplying both sides by $-1$]

Thus, the solution set is $[2, \infty)$.

On number line, it can be represented as




Here, the dark portion on the number line represents the solution of inequality.

# TOPIC PRACTICE 1

## OBJECTIVE TYPE QUESTIONS

**1** Which of the following is/are the examples of numerical inequalities?

I. $3 < 5$ II. $7 > 5$

III. $x < 5$ IV. $y > 2$

V. $x \ge 3$

(a) I, II (b) III, IV

(c) IV (d) All of these

**2** The solution set of the inequality $4x + 3 < 6x + 7$ is

(a) $[-2, \infty)$ (b) $(-\infty, -2)$

(c) $(-2, \infty)$ (d) None of these

**3** The set of real $x$ satisfying the inequality $\frac{5-2x}{3} \le \frac{x}{6} - 5$ is

(a) $(-\infty, 8)$ (b) $(8, \infty)$

(c) $[8, \infty)$ (d) $(-\infty, 8]$

**4** If $3x + 8 > 2$, then which of the following is true?

(a) $x \in \{-1, 0, 1, 2, \ldots\}$, when $x$ is an integer

(b) $x \in [-2, \infty)$, when $x$ is a real number

(c) Both (a) and (b)

(d) None of the above

**5** The solution set of the inequality

$4x + 3 < 5x + 7 \ \forall \ x \in R$ is

(a) $(-4, \infty)$ (b) $[-4, \infty)$

(c) $(4, \infty)$ (d) $[4, \infty)$

## VERY SHORT ANSWER Type Questions

**6** Solve $-12x > 30$, when

(i) $x$ is a natural number.

(ii) $x$ is an integer. **[NCERT]**

**7** Solve the inequality for real $x$,

$4x + 3 < 6x + 7$. **[NCERT]**

**8** Solve the inequality $5x - 3 < 3x + 1$, when $x$ is an integer.

**9** Solve the inequality $3(x - 1) \le 2(x - 3)$ for real $x$.

## SHORT ANSWER Type Questions

**10** Solve $3x + 8 > 2$, when

(i) $x$ is an integer.

(ii) $x$ is a real number. **[NCERT]**

**11** Solve the inequality $2(2x + 3) - 10 < 6(x - 2)$ for real $x$. **[NCERT]**

**12** Solve the inequality $5x - 1 > 3x + 7$ and show the graph of the solution on number line.

**13** Solve the inequality $\frac{x}{3} > \frac{x}{2} + 1$ and show the graph of the solution on number line.

## LONG ANSWER Type Questions

**14** Solve $37 - (3x + 5) \ge 9x - 8(x - 3)$. Show the graph of the solution on number line. **[NCERT]**

**15** Solve the inequality $\frac{x}{2} \ge \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$ and show the graph of the solution on number line.

**[NCERT]**

# HINTS & ANSWERS

**1.** (a) $3 < 5$; $7 > 5$ are the examples of numerical inequalities while $x < 5$; $y > 2$; $x \ge 3$ are the example of literal inequalities.

**2.** (c) We have, $4x + 3 < 6x + 7$

or $-2x < 4$ or $x > -2$

Hence, the solution set is $(-2, \infty)$.

**3.** (c) We have, $\frac{5-2x}{3} \le \frac{x}{6} - 5$

or $-5x \le -40$, i.e. $x \ge 8$

$x \in [8, \infty)$.

**4.** (a) We have, $3x + 8 > 2$

On adding $-8$ both sides,

$\Rightarrow \quad 3x > -6$

On dividing by 3 both sides,

$\Rightarrow \quad x > -2$

(i) When $x$ is an integer, the solution of the given inequality is $\{-1, 0, 1, 2 ....\}$.

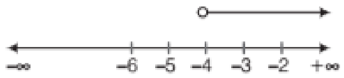
(ii) When $x$ is a real number, the solution of the given inequality is $(-2, \infty)$. *i.e.*, all the numbers lying between $-2$ and $\infty$ but $-2$ and $\infty$ are not included.

**5.** (a) We have, $4x + 3 < 5x + 7$

On adding $-5x - 3$ both sides, we get

$4x - 5x < 7 - 3 \Rightarrow -x < 4 \Rightarrow x > -4$

With the help of number line, we can easily look for the numbers greater than $-4$.




$\therefore$ Solution set is $(-4, \infty)$ *i.e.*, all the numbers lying between $-4$ and $\infty$ but $-4$ and $\infty$ are not included as $x > -4$.

**6.** $-12x > 30 \Rightarrow x < -\dfrac{30}{12}$

**Ans.** (i) No solution

(ii) $\{..., -4, -3\}$

**7.** Subtracting $(4x + 7)$ from both sides of $4x + 3 < 6x + 7$.

**Ans.** $(-4, \infty)$

**8.** Subtracting $(3x - 3)$ from both sides of $5x - 3 < 3x + 1$.

**Ans.** $\{..., -4, -3, -2, -1, 0, 1\}$

**9.** Solve as Example 4.

**Ans.** $(-\infty, -3]$

**10.** Solve as Example 3.

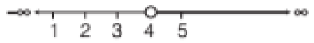
**Ans.** (i) $\{-1, 0, 1, 2, 3, ...\}$

(ii) $(-2, \infty)$

**11.** $(4, \infty)$

**12.** Subtracting $(3x - 1)$ from both sides of $5x - 1 > 3x + 7$.

**Ans.** $(4, \infty)$

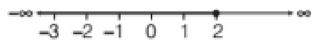



**13.** Solve as Example 5, part (ii).

**Ans.** $(-\infty, -6)$


**14.** Solve as Example 5, part (iii).

**Ans.** $(-\infty, 2]$




**15.** Solve as Example 5, part (iv).

**Ans.** $\left[-\dfrac{2}{7}, \infty\right)$


# |TOPIC 2|

## System of Inequalities in One Variable and Their Solutions

Two or more inequalities taken together comprise a system of inequalities and the solution of the system of inequalities are the solutions common to all the inequalities comprising the system.

e.g. $x = 10$ is the solution of the system of inequalities

$$4x + 3 \le 91$$

and $\quad 2x \ge x + 8$

### Solution of System of Linear Inequalities in One Variable

We know that, the solution set of a linear inequality in one variable is the set of all points on the number line satisfying the given inequality.

Therefore, the solution set of a system of linear inequalities in one variable is defined as the **intersection of the solution set** of the linear inequalities in the system.

e.g. If the solution sets of linear inequalities in the system are $(-\infty, 5]$ and $[5, \infty)$, then the solution of system of linear inequalities in one variable is 5 only. Because, if we represent the solution sets on the number line, we see that the value which are common to both is 5 only.


The process of finding solution of system of linear inequalities of different types are given below.

## |TYPE I|

### WHEN TWO SEPARATE LINEAR INEQUALITIES ARE GIVEN

If the given system of inequalities comprise by two separate linear inequalities, then to solve these we use the following working steps

*Step* I Solve each inequality separately and obtain their solution sets.

*Step* II Represent the solution sets on a number line and then find the values of the variable which are common to them.

*Or*

Find the intersection of the solution sets obtained in step I.

**EXAMPLE |1|** Solve the following system of inequalities $2x-3<7$ and $2x>-4$. Also, represent the solution graphically on the number line.

***Sol.*** We have the following inequalities,

$$2x-3<7 \quad \text{...(i)}$$

and $$2x>-4 \quad \text{...(ii)}$$

From inequality (i), we have $2x-3<7$

$\Rightarrow \quad 2x-3+3<7+3$ [adding 3 on both sides]

$\Rightarrow \quad 2x<10$

$\Rightarrow \quad \frac{2x}{2}<\frac{10}{2}$ [dividing both sides by 2]

$\Rightarrow \quad x<5$

$\therefore$ The solution set is $(-\infty, 5)$.

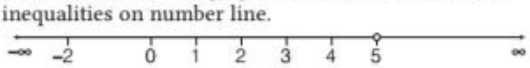
From inequality (ii), we have $2x>-4$

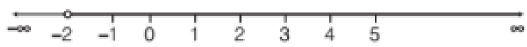
$\frac{2x}{2}>\frac{-4}{2}$ [dividing both sides by 2]

$\Rightarrow \quad x>-2$

$\therefore$ The solution set is $(-2, \infty)$.

Now, let us draw the graphs of the solutions of both inequalities on number line.




It can be seen that the values of $x$, which are common to both are lying in the interval $(-2, 5)$.

Hence, the solution set of given system of inequations is $(-2, 5)$ and this can be represented graphically on the number line as


**EXAMPLE |2|** Solve the following system of inequalities and represent the solution graphically on the number line. **[NCERT]**

$$5(2x-7)-3(2x+3)\le 0,\ 2x+19\le 6x+47$$

***Sol.*** We have the following inequalities

$$5(2x-7)-3(2x+3)\le 0 \quad \text{...(i)}$$

and $$2x+19\le 6x+47 \quad \text{...(ii)}$$

From inequality (i), we get

$5(2x-7)-3(2x+3)\le 0$

$\Rightarrow \quad 10x-35-6x-9\le 0 \Rightarrow 4x-44\le 0$

$\Rightarrow \quad 4x-44+44\le 44$ [adding 44 both sides]

$\Rightarrow \quad 4x\le 44$

$\Rightarrow \quad x\le 11$ ...(iii)

[dividing both sides by 4]

$\therefore$ The solution set is $(-\infty, 11]$.

From inequality (ii), we get

$2x+19\le 6x+47$

$\Rightarrow \quad 2x+19-2x\le 6x+47-2x$

[subtracting $2x$ from both sides]

$\Rightarrow \quad 19\le 4x+47$

$\Rightarrow \quad 19-47\le 4x+47-47$

[subtracting 47 from both sides]

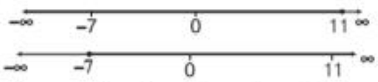
$\Rightarrow \quad -28\le 4x$ or $4x\ge -28$

$\Rightarrow \quad \frac{4x}{4}\ge\frac{-28}{4}$ [dividing both sides by 4]

$\Rightarrow \quad x\ge -7$

$\therefore$ The solution set is $[-7, \infty)$.

Now, let us draw the graphs of the solutions of both inequalities on number line.




It can be seen that the values of $x$, which are common to both are lying in the interval $[-7, 11]$.

Hence, the solution set of given system of inequations is $[-7, 11]$ and this can be represented graphically on the number line as


**EXAMPLE |3|** Solve the following system of linear inequalities and represent the solution graphically on the number line.

$$2(2x+3)-10<6(x-2) \text{ and } \frac{2x-3}{4}+6\ge 2+\frac{4x}{3}$$

Firstly, convert the given inequalities in simplest form, then solve each inequality separately and take the common solution.

***Sol.*** Given system of linear inequalities is

$$2(2x+3)-10<6(x-2) \quad \text{...(i)}$$

and $$\frac{2x-3}{4}+6\ge 2+\frac{4x}{3} \quad \text{...(ii)}$$

From inequality (i), we get

$2(2x+3)-10<6(x-2)$

$\Rightarrow \quad 4x+6-10<6x-12 \Rightarrow 4x-4<6x-12$

$\Rightarrow \quad 4x-4+4<6x-12+4$ [adding 4 both sides]

$\Rightarrow \quad 4x<6x-8$

$\Rightarrow \quad 4x-6x<6x-8-6x$

[subtracting $6x$ from both sides]

$\Rightarrow \quad -2x<-8$

$\Rightarrow \quad 2x>8$ [dividing both sides by $-1$, then inequality sign will also change]

$\Rightarrow \quad \frac{2x}{2}>\frac{8}{2}$ [dividing both sides by 2]

$\therefore \quad x>4$ ...(iii)

Thus, any value of $x$ greater than 4 satisfies the inequality.

$\therefore$ The solution set is $(4, \infty)$.

The representation of solution of inequality (i) is


From inequality (ii), we get

$$\frac{2x-3}{4}+6 \geq 2+\frac{4x}{3}$$

$\Rightarrow \quad \frac{2x-3+24}{4} \geq \frac{6+4x}{3}$

$\Rightarrow \quad \frac{2x+21}{4} \geq \frac{6+4x}{3}$

$\Rightarrow \quad 3(2x+21) \geq 4(6+4x)$

[multiplying both sides by LCM $(4, 3)=12$]

$\Rightarrow \quad 6x+63 \geq 24+16x$

$\Rightarrow \quad 6x+63-63 \geq 24+16x-63$

[subtracting 63 from both sides]

$\Rightarrow \quad 6x \geq 16x-39$

$\Rightarrow \quad 6x-16x \geq 16x-39-16x$

[subtracting $16x$ from both sides]

$\Rightarrow \quad -10x \geq -39$

$\Rightarrow \quad 10x \leq 39$

[multiplying both sides by $-1$, the inequality sign will also change]

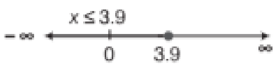
$\Rightarrow \quad \frac{10x}{10} \leq \frac{39}{10}$ [dividing both sides by 10]

$\Rightarrow \quad x \leq 3.9$ ...(iv)

Thus, any value of $x$ less than or equal to 3.9 satisfies the inequality.

$\therefore$ The solution set is $(-\infty, 3.9]$.

Its representation on number line is,




We see that there is no common value of $x$, which satisfies both inequalites (iii) and (iv).

Hence, the given system of inequalities has no solution set.

## |TYPE 2|

### WHEN INEQUALITIES OF THE FORM $a \leq \frac{cx+d}{e} \leq b$, WHERE $a, b, c, d \in R$

This type of inequalities will be formed by combining the inequalities $a \leq \frac{cx+d}{e}$ and $\frac{cx+d}{e} \leq b$.

To solve such type of inequalities, make the middle term free from constant (i.e. write the given inequalities as $f \leq x \leq g$, where $f$ and $g$ are some real numbers by using the rule of addition, subtraction, multiplication, division in each term of given inequalities.

**EXAMPLE |4|** Solve the inequalities $6 \leq -3(2x-4) < 12$. **[NCERT]**

***Sol.*** We have, $6 \leq -3(2x-4)<12$ or $6 \leq -6x+12<12$

On subtracting 12 from each term, we get

$$6-12 \leq -6x+12-12<12-12$$

$\Rightarrow \quad -6 \leq -6x<0$

On dividing each term by $-6$, we get $\frac{-6}{-6} \geq \frac{-6x}{-6} > \frac{0}{-6}$

[while dividing each term by the same negative number, then sign of inequalities will get change]

$\Rightarrow \quad 1 \geq x>0$

which can be written as $0<x \leq 1$.

Hence, solution set of given system of inequations is $(0, 1]$.

**EXAMPE |5|** Solve the inequalities $-3 \leq 4-\frac{7x}{2} \leq 18$. **[NCERT]**

***Sol.*** We have, $-3 \leq 4-\frac{7x}{2} \leq 18$

On subtracting 4 from each term, we get

$-3-4 \leq 4-\frac{7x}{2}-4 \leq 18-4 \Rightarrow -7 \leq -\frac{7x}{2} \leq 14$

On multiplying each term by $\left(\frac{-2}{7}\right)$, we get

$$-7\left(-\frac{2}{7}\right) \geq \frac{-7}{2}x \times \left(\frac{-2}{7}\right) \geq 14 \times \left(\frac{-2}{7}\right)$$

[while multiplying each term by the same negative number, then the sign of inequalities will get change]

$\Rightarrow \quad 2 \geq x \geq -4$ or $-4 \leq x \leq 2$ or $x \in [-4, 2]$

Hence, solution set of given system of inequations is $[-4, 2]$.

**EXAMPLE |6|** Solve the linear inequalities

$$-15 < \frac{3(x-2)}{5} \leq 0.$$

**Sol.** We have, $-15 < \dfrac{3(x-2)}{5} \le 0$

$\Rightarrow \quad -15 \times 5 < \dfrac{3(x-2)}{5} \times 5 \le 0 \times 5$

[multiplying each term by 5]

$\Rightarrow \quad -75 < 3(x-2) \le 0$

$\Rightarrow \quad \dfrac{-75}{3} < \dfrac{3(x-2)}{3} \le \dfrac{0}{3}$

[dividing each term by 3]

$\Rightarrow \quad -25 < x - 2 \le 0$

$\Rightarrow \quad -25 + 2 < x - 2 + 2 \le 0 + 2$

[adding 2 each term]

$\Rightarrow \quad -23 < x \le 2$

$\therefore \quad x \in (-23, 2]$

Hence, solution set of the given system of inequations is $(-23, 2]$.

## |TYPE 3|

### WHEN INEQUALITY OF THE FORM $\dfrac{ax+b}{cx+d} \gtreqless k$

If the given inequality is of the form

$$\frac{ax+b}{cx+d} > k \quad \text{or} \quad \frac{ax+b}{cx+d} \ge k$$

or
$$\frac{ax+b}{cx+d} < k \quad \text{or} \quad \frac{ax+b}{cx+d} \le k,$$

where $k$ is a constant, then we solve these inequalities with the help of following working steps

*Step* I Collect all terms in the LHS of given inequality to make RHS zero and then reduce it any one of the form

$$\frac{px+q}{cx+d} > 0 \quad \text{or} \quad \frac{px+q}{cx+d} \ge 0$$

$$\text{or} \quad \frac{px+q}{cx+d} < 0 \quad \text{or} \quad \frac{px+q}{cx+d} \le 0$$

*Step* II Multiply the above inequality by square of denominator to reduce it in the form of product of two expressions, i.e. $(px+q)(cx+d) \gtreqless 0$.

*Step* III Now, use the result

(i) If product of two terms $< 0$, then both terms have opposite sign.

(ii) If product of two terms $> 0$, then both terms have same sign.

*Step* IV Now, solve both expression separately with suitable inequality sign and get solution sets.

*Step* V Take the union of above solution sets, which gives the solution of given inequality.

**EXAMPLE |7|** Solve the inequality $\dfrac{x-2}{x+5} > 2$. Also, represent the solution on the number line.

**Sol.** We have, $\dfrac{x-2}{x+5} > 2$

On subtracting 2 from both sides, we get

$$\frac{x-2}{x+5} - 2 > 0 \Rightarrow \frac{x-2-2(x+5)}{x+5} > 0$$

$$\Rightarrow \frac{x-2-2x-10}{x+5} > 0 \Rightarrow \frac{-(x+12)}{x+5} > 0 \Rightarrow \frac{x+12}{x+5} < 0$$

[multiplying both sides by $-1$]

On multiplying both sides by $(x+5)^2$, we get

$$\frac{(x+12)}{(x+5)} \times (x+5)^2 < 0 \times (x+5)^2 \Rightarrow (x+12)(x+5) < 0$$

So, the factors $(x+12)$ and $(x+5)$ have opposite signs.

$\therefore \quad (x+12) > 0$ and $(x+5) < 0$ ...(i)

or $\quad (x+12) < 0$ and $(x+5) > 0$ ...(ii)

From inequality (i), we get

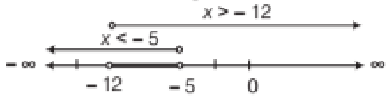
$x + 12 > 0$ and $x + 5 < 0$

$\Rightarrow \quad x + 12 - 12 > -12$ and $x + 5 - 5 < 0 - 5$

[subtracting 12 from first inequality and 5 from second]

$\Rightarrow \quad x > -12$ and $x < -5$ ...(iii)

On number line, it can be represented as




$\therefore \quad -12 < x < -5$ ...(iv)

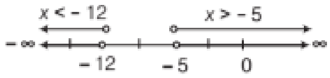
From inequality (ii), we get

$(x+12) < 0$ and $(x+5) > 0$

$x + 12 - 12 < 0 - 12$ and $x + 5 - 5 > -5$

$\Rightarrow \quad x < -12$ and $x > -5$

On number line, it can be represented as




Since, no common point occurs, so it has no solution.

Hence, the solution set of the given inequality is $-12 < x < -5$ i.e. $(-12, -5)$.

**EXAMPLE |8|** Solve the inequality $\dfrac{x-4}{x+2} \le 2$. Also, represent the solution on number line.

**Sol.** We have, $\dfrac{x-4}{x+2} \le 2$

$\Rightarrow \quad \dfrac{x-4}{x+2} - 2 \le 2 - 2$ [subtracting 2 from both sides]

$\Rightarrow \quad \dfrac{x-4}{x+2} - 2 \le 0$

$$\Rightarrow \quad \frac{x-4-2(x+2)}{x+2} \le 0$$

$$\Rightarrow \quad \frac{x-4-2x-4}{x+2} \le 0$$

$$\Rightarrow \quad \frac{-x-8}{x+2} \le 0, \text{ here } x+2 \ne 0; \text{ i.e. } x \ne -2$$

$$\Rightarrow \quad \frac{x+8}{x+2} \ge 0 \quad \text{[multiplying both sides by } -1]$$

$$\Rightarrow \quad \frac{x+8}{x+2} \times (x+2)^2 \ge 0 \times (x+2)^2$$

[multiplying both sides by $(x+2)^2$]

$$\Rightarrow \quad (x+8)(x+2) \ge 0$$

The factors $(x+8)$ and $(x+2)$ have the same signs.

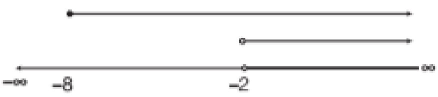
***Case* I When both are non-negative**

In this case, we have

$x+8 \ge 0$ and $x+2 > 0$ [$\because x+2 \ne 0$ because for $x=-2$, denominator of given inequality becomes 0]

$\Rightarrow \quad x \ge -8$ and $x > -2$

On number line, these can be represented as




Clearly, the common values of $x$ are lying in the interval $(-2, \infty)$.

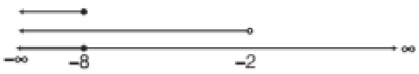
$\therefore$ The solution set is $(-2, \infty)$.

***Case* II When both are non-positive**

In this case, we have

$x+8 \le 0$ and $x+2 < 0 \Rightarrow x \le -8$ and $x < -2$

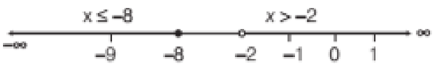
On number line, these can be represented as




Clearly, the common values of $x$ are lying in the interval $(-\infty, -8]$

$\therefore$ The solution set is $(-\infty, -8]$.

Hence, the required solution set is $(-\infty, -8] \cup (-2, \infty)$ and this can be represented on the number line as




**EXAMPLE |9|** Solve the inequality $\frac{-3x+10}{x+1} > 0$. Also, represent the solution on the number line.

***Sol.*** We have, $\frac{-3x+10}{x+1} > 0$

$$\Rightarrow \quad \frac{-3x+10}{x+1} \times (x+1)^2 > 0 \times (x+1)^2$$

[multiplying both sides by $(x+1)^2$]

$$\Rightarrow \quad (-3x+10)(x+1) > 0$$

$\Rightarrow$ The factors $(-3x+10)$ and $(x+1)$ have the same signs.

***Case* I When both are positive**

In this case, we have

$$-3x+10 > 0 \text{ and } x+1 > 0$$

$$\Rightarrow \quad 10 > 3x \text{ and } x > -1$$

$$\Rightarrow \quad 3x < 10 \text{ and } x > -1$$

$$\Rightarrow \quad x < \frac{10}{3} \text{ and } x > -1$$

$$\Rightarrow \quad -1 < x < \frac{10}{3}$$

$\therefore$ The solution set is $\left(-1, \frac{10}{3}\right)$.

***Case* II When both are negative**

In this case, we have

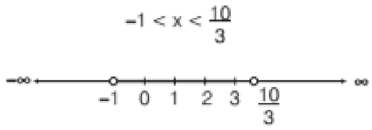
$$-3x+10 < 0 \text{ and } x+1 < 0$$

$$\Rightarrow \quad 3x > 10 \text{ and } x < -1$$

$$\Rightarrow \quad x > \frac{10}{3} \text{ and } x < -1, \text{ which is impossible}$$

[$\because$ system of inequalities have no common solution]

Hence, the required solution set is $\left(-1, \frac{10}{3}\right)$ and this can be represented on the number line as




**EXAMPLE |10|** Solve the inequality $\frac{x+3}{x+4} \ge 1$ and show the solution on the number line. **[NCERT Exemplar]**

***Sol.*** We have, $\frac{x+3}{x+4} \ge 1$

$$\Rightarrow \quad \frac{x+3}{x+4} - 1 \ge 1 - 1$$

[subtracting 1 from both sides]

$$\Rightarrow \quad \frac{x+3-x-4}{x+4} \ge 0$$

$$\Rightarrow \quad \frac{-1}{x+4} \ge 0$$

$$\Rightarrow \quad x+4 \le 0, \; x+4 \ne 0$$

[multiplying both sides by $-(x+4)^2$]

$$\Rightarrow \quad x+4-4 \le 0-4, \; x \ne -4$$

[subtracting 4 from both sides]

$$x \le -4, \; x \ne -4$$

$\therefore$ The solution set is $(-\infty, -4)$.

On number line, it can be represented as

## | TYPE 4 |

### WHEN INEQUALITY OF THE FORM $|ax+b| \gtreqless k$, WHERE $a, b, k$ ARE REAL NUMBERS AND $a \neq 0, k > 0$

If the given inequality is any of the form $|ax+b| < k$ or $|ax+b| > k$ or $|ax+b| \geq k$ or $|ax+b| \leq k$, where $a \neq 0$ and $k > 0$, then we solve the inequality with the help of following results.

If $a > 0$ is any real number, then

(i) $|x| < a \Rightarrow -a < x < a$

$|x| \leq a \Rightarrow -a \leq x \leq a$

(ii) $|x| > a \Rightarrow x > a$ or $x < -a$

$|x| \geq a \Rightarrow x \geq a$ or $x \leq -a$

**Note**

In the above results, $x$ can be any real number or any linear expression.

### Expansion of Modulus Function

$$|x-a| = \begin{cases} x-a, & \text{if } x \geq a \\ -(x-a), & \text{if } x < a \end{cases}$$ , where $a$ is any real number.

**EXAMPLE |11|** Solve $|3x-2| \leq \frac{1}{2}$.

**Sol.** Given, $|3x-2| \leq \frac{1}{2}$

$\therefore \quad \Rightarrow \frac{-1}{2} \leq (3x-2) \leq \frac{1}{2}$ $\quad [\because |x| \leq a \Rightarrow -a \leq x \leq a]$

$\Rightarrow \quad -\frac{1}{2}+2 \leq 3x-2+2 \leq \frac{1}{2}+2$ $\quad$ [adding 2 each term]

$\Rightarrow \quad \frac{3}{2} \leq 3x \leq \frac{5}{2}$

$\Rightarrow \quad \frac{3}{2} \times \frac{1}{3} \leq \frac{3x}{3} \leq \frac{5}{2} \times \frac{1}{3}$ [dividing each term by 3]

$\Rightarrow \quad \frac{1}{2} \leq x \leq \frac{5}{6}$ i.e. $x \in \left[\frac{1}{2}, \frac{5}{6}\right]$

Hence, the required solution set is $\left[\frac{1}{2}, \frac{5}{6}\right]$.

**EXAMPLE |12|** Solve $|x-2| \geq 6$.

**Sol.** We have, $\quad |x-2| \geq 6$

$\Rightarrow \quad x-2 \geq 6$ or $x-2 \leq -6$

$[\because |x| \geq a \Rightarrow x \geq a$ or $x \leq -a]$

$\Rightarrow \quad x \geq 8$ or $x \leq -4$

$\Rightarrow \quad x \in [8, \infty)$ or $x \in (-\infty, 4]$

$\Rightarrow \quad x \in [8, \infty) \cup (-\infty, 4]$

Hence, the required solution set is $(-\infty, 4] \cup [8, \infty)$.

**EXAMPLE |13|** Solve graphically for $x$, $|x-1| \leq 5$, $|x| \geq 2$.

Solve the inequalities separately and then take the intersection of their solution sets.

**Sol.** We have the following inequalities

$|x-1| \leq 5$ $\quad$ ...(i)

and $\quad |x| \geq 2$ $\quad$ ...(ii)

From inequality (i), we have

$-5 \leq x-1 \leq 5$ $\quad [\because |x| \leq a \Rightarrow -a \leq x \leq a]$

$\Rightarrow \quad -5+1 \leq x \leq 5+1$ $\quad$ [adding 1 each term]

$\Rightarrow \quad -4 \leq x \leq 6$

$\Rightarrow \quad x \in [-4, 6]$

$\therefore$ The solution set is $[-4, 6]$.

From inequality (ii), we have

$|x| \geq 2$

$\Rightarrow \quad x \geq 2$ or $x \leq -2$

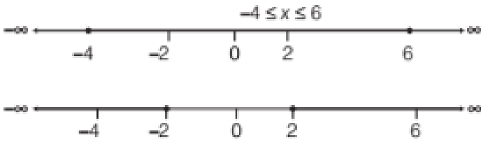
$[\because |x| \geq a \; x \geq a$ or $x \leq -a]$

$\Rightarrow \quad x \in [2, \infty)$ or $(-\infty, -2]$

$\Rightarrow \quad x \in (-\infty, -2] \cup [2, \infty)$

$\therefore$ The solution set is $(-\infty, -2] \cup [2, \infty)$.

Now, let us draw the graphs of solutions of both inequality.




Clearly, the common values of $x$ are lying in the interval $[-4, -2]$ or $[2, 6]$.

Hence, the required solution set is $[-4, -2] \cup [2, 6]$.

## | TYPE 5 |

### SOLUTION OF AN INEQUALITY, WHICH CONTAIN THE TERM(S) OF THE FORM $|x \pm a|$ AND $|ax \pm b|$

If the given inequality contain the term(s) of the form $|x \pm a|$ or $|ax+b|$, then proceed as follows

*Step* I Put each modulus term equal to zero and find the corresponding value of $x$. These value of $x$ are called the **critical points**.

e.g. If the given inequality involves the terms

$|x-a|$ and $|x-b|$ then put

$|x-a| = 0$ and $|x-b| = 0$

$\Rightarrow \quad x-a = 0$ and $x-b = 0$

$\Rightarrow \quad x = a$ and $x = b$

*Step* II Divide the whole real line into disjoint intervals with the help of critical points.

e.g. If $x=a$ and $x=b$ are critical points, such that $a<b$, then divide the real line as


***Step* III** Consider the intervals obtained in step II and solve the given inequality in each interval.

***Step* IV** Take the union of solution sets, obtained in each case.

This gives the required solution set.

**EXAMPLE |14|** Solve $|x-1|+|x-2|\geq 4$.

***Sol.*** We have, $|x-1|+|x-2|\geq 4$

Put $\quad x-1=0 \Rightarrow x=1$

and $\quad x-2=0 \Rightarrow x=2$

Thus, $x=1$ and 2 are critical points, so we will consider three intervals $(-\infty, 1)$, $[1, 2)$ and $[2, \infty)$.

***Case* I** When $-\infty<x<1$

Then, $|x-1|=-(x-1)$ and $|x-2|=-(x-2)$

$$\left[\because |x-a|=\begin{cases} x-a, & \text{if } x\geq a \\ -(x-a), & \text{if } x<a\end{cases}\right]$$

$\therefore \quad |x-1|+|x-2|\geq 4$

$\Rightarrow \quad -(x-1)-(x-2)\geq 4$

$\Rightarrow -x+1-x+2\geq 4 \Rightarrow -2x+3\geq 4$

$\Rightarrow \quad -2x\geq 1$ [subtracting 3 from both sides]

$\Rightarrow \quad x\leq -\dfrac{1}{2}$ [dividing both sides by −2]

$\therefore$ Solution set of given inequality in this case is

$$-\infty<x\leq -\frac{1}{2} \qquad \text{...(i)}$$

***Case* II** When $1\leq x<2$

Then, $\quad |x-1|=x-1$

and $\quad |x-2|=-(x-2)$

$\therefore \quad |x-1|+|x-2|\geq 4$

$\Rightarrow \quad x-1-x+2\geq 4$

$\Rightarrow 1\geq 4$, which is absurd.

So, in this case, given inequality has no solution.

***Case* III** When $2\leq x<\infty$

Then, $|x-1|=x-1$ and $|x-2=x-2$

$\therefore \quad |x-1|+|x-2|\geq 4$

$\Rightarrow \quad x-1+x-2\geq 4$

$\Rightarrow \quad 2x-3\geq 4$

$\Rightarrow \quad 2x\geq 7$ [adding 3 both sides]

$\Rightarrow \quad x\geq \dfrac{7}{2}$ [dividing both sides by 2]

Here, $2\leq x<\infty$.

$\therefore$ Solution set of given inequality in this case is

$$\frac{7}{2}\leq x<\infty \qquad \text{...(ii)}$$

On combining inequalities (i) and (ii), we get

$$-\infty<x\leq -\frac{1}{2} \text{ and } \frac{7}{2}\leq x<\infty$$

i.e. $\quad x\in\left(-\infty, -\dfrac{1}{2}\right]\cup\left[\dfrac{7}{2}, \infty\right)$

**EXAMPLE |15|** Solve the inequality $\dfrac{|x|-1}{|x|-2}\geq 0$, $x\in R$ and $x\neq \pm 2$.

***Sol.*** We have, $\dfrac{|x|-1}{|x|-2}\geq 0$

Now, put $|x|=0 \Rightarrow x=0$

Thus, $x=0$ is the critical point. So, we will consider two intervals $(-\infty, 0)$ and $[0, \infty)$.

***Case* I** When $x\in(-\infty, 0)$, i.e. $-\infty<x<0$

In this case, we have

$$\frac{-x-1}{-x-2}\geq 0 \quad \left[\because |x|=\begin{cases} x, & \text{if } x\geq 0 \\ -x, & \text{if } x<0\end{cases}\right]$$

$\Rightarrow \quad \dfrac{-(x+1)}{-(x+2)}\geq 0 \Rightarrow \dfrac{x+1}{x+2}\geq 0$

Now, multiplying both sides by $(x+2)^2$, we get

$$(x+1)(x+2)\geq 0$$

So, the factors $(x+1)$ and $(x+2)$ have the same sign.

Then, we have

$$x+1\geq 0 \text{ and } x+2>0 \qquad \text{...(i)}$$

or $$x+1\leq 0 \text{ and } x+2<0 \qquad \text{...(ii)}$$

[$\because x+2\neq 0$, as $x\neq -2$]

From inequality (i), we get

$$x\geq -1 \text{ and } x>-2$$

$\Rightarrow \quad x\geq -1$ [common values of $x$]

Thus, the common values of $x$ are lying in the interval $[-1, 0)$.

[$\because$ we are considering the case, where $-\infty<x<0$]

$\therefore$ The solution set is $[-1, 0)$.

From inequality (ii), we have

$$x\leq -1 \text{ and } x<-2$$

$\Rightarrow \quad x<-2$ [common values of $x$]

Thus, the common values of $x$ are lying in the interval $(-\infty, -2)$

$\therefore$ The solution set is $(-\infty, -2)$.

Hence, $x\in(-\infty, -2)\cup[-1, 0)$

***Case* II** When $x\in[0, \infty)$, i.e. $0\leq x<\infty$

In this case, we have

$$\frac{x-1}{x-2}\geq 0 \quad \left[\because |x|=\begin{cases} x, & \text{if } x\geq 0 \\ -x, & \text{if } x<0\end{cases}\right]$$

On multiplying both sides by $(x-2)^2$, we get

$$(x-1)(x-2) \geq 0$$

So, the factors $(x-1)$ and $(x-2)$ have the same sign.

Then we have,

$$(x-1) \geq 0 \text{ and } (x-2) > 0 \quad \text{...(iii)}$$

or $$(x-1) \leq 0 \text{ and } (x-2) < 0 \quad \text{...(iv)}$$

[$\because x-2 \neq 0$ as $x \neq 2$]

From inequality (iii), we get

$x \geq 1$ and $x > 2$

$\Rightarrow$ $x > 2$ [common values of $x$]

$\Rightarrow$ $x \in (2, \infty)$

$\therefore$ The solution set is $(2, \infty)$.

From inequality (iv), we get

$x \leq 1$ and $x < 2$

$\Rightarrow$ $x \leq 1$ [common values of $x$]

$\Rightarrow$ $x \in [0, 1]$

[$\because$ we are considering the case, where $0 \leq x < \infty$]

$\therefore$ The solution set is $[0, 1]$

Hence, $x \in [0, 1] \cup (2, \infty)$

Now, from cases I and II, we conclude that

$$x \in (-\infty, -2) \cup [-1, 0) \cup [0, 1] \cup (2, \infty)$$

i.e. $$x \in (-\infty, -2) \cup [-1, 1] \cup (2, \infty)$$

Hence, the required solution set is

$$(-\infty, -2) \cup [-1, 1] \cup (2, \infty)$$

**Alternate Method**

We have, $$\frac{|x|-1}{|x|-2} \geq 0$$

Let $y = |x|$, then we have $\frac{y-1}{y-2} \geq 0$

On multiplying both sides by $(y-2)^2$, we get

$$(y-1)(y-2) \geq 0$$

So, the factors $(y-1)$ and $(y-2)$ have the same signs.

Then, we have, $y-1 \geq 0$ and $y-2 > 0$ ...(i)

or $y-1 \leq 0$ and $y-2 < 0$ ...(ii)

[$\because y-2 \neq 0$ as $y = |x| \neq 2$]

From inequality (i), we get

$y \geq 1$ and $y > 2$

From inequality (ii), we get

$y \leq 1$ and $y < 2$

$\Rightarrow$ $y \leq 1$ [common values of $y$]

Thus, $y \leq 1$ or $y > 2$

$\Rightarrow$ $|x| \leq 1$ or $|x| > 2$ [$\because y = |x|$]

$\Rightarrow$ $(-1 \leq x \leq 1)$ or $(x > 2$ or $x < -2)$

$\Rightarrow$ $x \in [-1, 1]$ or $x \in (-\infty, -2) \cup (2, \infty)$

$\Rightarrow$ $x \in [-1, 1] \cup (-\infty, -2) \cup (2, \infty)$

Hence, required solution set is

$$(-\infty, -2) \cup [-1, 1] \cup (2, \infty)$$

**EXAMPLE |16|** Solve for $x$, $\frac{|x+3|+x}{x+2} > 1$. **[NCERT Exemplar]**

***Sol.*** We have, $\frac{|x+3|+x}{x+2} > 1 \Rightarrow \frac{|x+3|+x}{x+2} - 1 > 0$

$$\Rightarrow \frac{|x+3|+x-x-2}{x+2} > 0 \Rightarrow \frac{|x+3|-2}{x+2} > 0$$

Put $x+3=0 \Rightarrow x=-3$

$\therefore$ $x=-3$ is a critical point.

So, here we consider two intervals $(-\infty, -3)$ and $[-3, \infty)$.

***Case* I** When $-3 \leq x < \infty$, then $|x+3| = (x+3)$

$$\therefore \frac{|x+3|-2}{x+2} > 0$$

$$\Rightarrow \frac{x+3-2}{x+2} > 0 \Rightarrow \frac{x+1}{x+2} > 0$$

$$\Rightarrow \frac{(x+1)(x+2)^2}{(x+2)} > 0 \times (x+2)^2$$

[multiplying both sides by $(x+2)^2$]

$$\Rightarrow (x+1)(x+2) > 0$$

Product of $(x+1)$ and $(x+2)$ will be positive, if both are of same signs.

$\therefore$ $(x+1) > 0$ and $(x+2) > 0$

or $(x+1) < 0$ and $(x+2) < 0$

$\Rightarrow$ $(x > -1$ and $x > -2)$

or $(x < -1$ and $x < -2)$

Thus, $-1 < x < \infty$ or $-\infty < x < -2$

But, here $-3 \leq x < \infty$

$\therefore$ $-1 < x < \infty$ or $-3 \leq x < -2$

Hence, solution set in this case is

$$x \in [-3, -2) \cup (-1, \infty)$$

***Case* II** When $x < -3$, then $|x+3| = -(x+3)$

$$\therefore \frac{|x+3|-2}{x+2} > 0 \Rightarrow \frac{-x-3-2}{x+2} > 0$$

$$\Rightarrow \frac{-(x+5)}{x+2} > 0 \Rightarrow \frac{x+5}{x+2} < 0$$

$$\Rightarrow \frac{(x+5)(x+2)^2}{x+2} < 0 \times (x+2)^2$$

Product of $(x+5)$ and $(x+2)$ will be negative, if both are of opposite sign.

$\therefore$ $(x+5) > 0$ and $(x+2) < 0$

or $(x+5) < 0$ and $(x+2) > 0$

$\Rightarrow$ $x > -5$ and $x < -2$

or $x < -5$ and $x > -2$ [not possible]

Thus, $-5 < x < -2$. i.e. solution set in the case is $(-5, -2)$.

On combining cases I and II, we get the required solution set of given inequality, which is

$$(-5, -2) \cup (-1, \infty).$$

# TOPIC PRACTICE 2

## OBJECTIVE TYPE QUESTIONS

**1** The solution set of the inequality $\frac{x+3}{x+4} \geq 1$, is

(a) $(-\infty, -4)$  (b) $(-\infty, 4)$
(c) $(-4, \infty)$  (d) $(4, \infty)$

**2** The solution set of the inequality $|3x-2| \leq \frac{1}{2}$, is

(a) $\left[\frac{1}{2}, \frac{3}{2}\right]$  (b) $\left[\frac{1}{2}, \frac{3}{4}\right]$
(c) $\left[\frac{1}{2}, \frac{5}{3}\right]$  (d) $\left[\frac{1}{2}, \frac{5}{6}\right]$

**3** Which of the following is the solution set of the inequality $\frac{x}{4} < \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$?

(a) $(4, \infty)$  (b) $(-\infty, 4)$
(c) $[4, \infty)$  (d) $(-\infty, 4]$

**4** $x$ and $b$ are real numbers. If $b > 0$ and $|x| > b$, then [NCERT Exemplar]

(a) $x \in (-b, \infty)$  (b) $x \in [-\infty, b)$
(c) $x \in (-b, b)$  (d) $x \in (-\infty, -b) \cup (b, \infty)$

**5** The solution set of the inequality $\frac{1}{2}\left(\frac{3x}{5} + 4\right) \geq \frac{1}{3}(x-6)$, is

(a) $(-\infty, -120)$  (b) $(-\infty, 120)$
(c) $(-\infty, 120]$  (d) $[-\infty, 120)$

## VERY SHORT ANSWER Type Questions

**6** Solve the inequalities $2 \leq 3x - 4 \leq 5$.

**7** Solve the inequality $\frac{1}{x-4} < 0$.

**8** Solve the following system of inequations.

$x + 2 > 0$ and $3x - 8 < 1$

**9** Solve the inequalities $-12 < 4 - \frac{3x}{-5} \leq 2$. [NCERT]

**10** Solve the inequalities $7 \leq \frac{(3x+11)}{2} \leq 11$. [NCERT]

## SHORT ANSWER Type I Questions

**11** Solve $|x-1| \leq 2$. [NCERT Exemplar]

**12** Solve $|x+3| \geq 10$. [NCERT Exemplar]

**13** Solve $|3x-7| > 2$. [NCERT Exemplar]

**14** Solve $|3-4x| \geq 9$. [NCERT Exemplar]

## SHORT ANSWER Type II Questions

**15** Solve the following system of inequalities and represent the solution graphically on number line.

$3x - 7 > 2(x-6)$ and $6 - x > 11 - 2x$

**16** Solve the following system of inequalities $7(2-3x) > 18 - 19x$ and $5 + 3x < 5x + 6$.

**17** Solve the inequality $\frac{x+8}{x-2} \geq 0$ and show the solution on number line.

**18** Solve for $x$, $\frac{1}{|x|-3} < 0$.

**19** Solve for $x$, $\frac{4}{x+1} \leq 3 \leq \frac{6}{x+1}$, $x > 0$. [NCERT Exemplar]

## LONG ANSWER Type Questions

**20** Solve the following system of linear inequalities

$$-2 - \frac{x}{4} \geq \frac{1+x}{3} \text{ and } 3 - x < 4(x-3).$$

**21** Solve the inequality $\frac{2x+5}{x-1} > 5$.

**22** Solve $1 \leq |x-2| \leq 3$. [NCERT Exemplar]

**23** Solve for $x$, $|x+1| + |x| > 3$. [NCERT Exemplar]

**24** Solve for $x$, $\frac{|x-2|-1}{|x-2|-2} \leq 0$. [NCERT Exemplar]

**25** Solve the following system of inequalities.

$$\frac{x}{2x+1} \geq \frac{1}{4} \text{ and } \frac{6x}{4x-1} < \frac{1}{2}$$ [NCERT Exemplar]

# HINTS & ANSWERS

**1.** (a) We have, $\frac{x+3}{x+4} \geq 1$

$\Rightarrow \quad \frac{x+3}{x+4} - 1 \geq 1 - 1$ [subtracting 1 from both sides]

$\Rightarrow \quad x < -4$

$\therefore$ The solution set is $(-\infty, -4)$.

**2.** (d) Given, $|3x-2| \leq \frac{1}{2}$

$\therefore \quad -\frac{1}{2} \leq (3x-2) \leq \frac{1}{2}$

[$\because |x| \leq a \Rightarrow -a \leq x \leq a$]

$\Rightarrow \quad \frac{3}{2} \leq 3x \leq \frac{5}{2}$ [adding 2 to each term]

$\Rightarrow \quad \frac{1}{2} \leq x \leq \frac{5}{6}$ [dividing each term by 3]

i.e., $\quad x \in \left[\frac{1}{2}, \frac{5}{6}\right]$

Hence, the required solution set is $\left[\frac{1}{2}, \frac{5}{6}\right]$.

**3.** (a) We have, $\frac{x}{4} < \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$

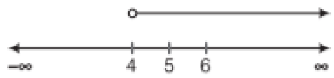
$\Rightarrow \quad 15x < 16x - 4$

Transferring the term $16x$ to LHS.

$15x - 16x < -4 \Rightarrow -x < -4$

On multiplying by $-1$ both sides, we get

$x > 4$




$\therefore$ Solution set is $(4, \infty)$.

**4.** (d) We have, $|x| > b, b > 0$

$\Rightarrow \quad x < -b$ and $x > b$

$\Rightarrow \quad x \in (-\infty, -b) \cup (b, \infty)$

**5.** (c) We have, $\frac{1}{2}\left(\frac{3x}{5} + 4\right) \geq \frac{1}{3}(x-6)$

$\Rightarrow \quad \frac{1}{2}\left(\frac{3x+20}{5}\right) \geq \frac{1}{3}(x-6)$

$\Rightarrow \quad 3(3x+20) \geq 10(x-6)$

$\Rightarrow \quad 9x + 60 \leq 10x - 60$

On adding $60 - 9x$ both sides, we get

$120 \geq x \Rightarrow x \leq 120$

$\therefore$ Solution set is $(-\infty, 120]$.

**6.** Solve as Example 1. **Ans.** $(-2, 3)$

**7.** Multiplying both sides by $(x-4)^2$. **Ans.** $(-\infty, 4)$

**8.** Solve as Example 4. **Ans.** $[2, 3]$

**9.** Solve as Example 5. **Ans.** $\left(\frac{-80}{3}, \frac{-10}{3}\right]$

**10.** Solve as Example 6. **Ans.** $\left[1, \frac{11}{3}\right]$

**11.** Use $|x| \leq a \Rightarrow -a \leq x \leq a$ and then solve as Example 11.

**Ans.** $[-1, 3]$

**12.** Use $|x| \geq a \Rightarrow x \geq a$ or $x \leq -a$ and then solved as Example 12.

**Ans.** $(-\infty, -13] \cup [7, \infty)$

**13.** Use $|x| > a \Rightarrow x > a$ or $x < -a$ and then solve as Example 12.

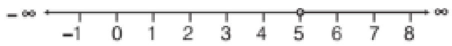
**Ans.** $\left(-\infty, \frac{5}{3}\right) \cup (3, \infty)$

**14.** Use $|x| \geq a \Rightarrow x \geq a$ or $x \leq -a$ and then solve as Example 12

**Ans.** $\left(-\infty \frac{-3}{2}\right] \cup [3, \infty)$

**15.** Solve as Example 2.

**Ans.** $(5, \infty)$




**16.** Solve as Example 2.

**Ans.** No solution exists.

**17.** Multiply both sides by $(x-2)^2$ and then solve as Example 8.

**Ans.** $(-\infty, -8] \cup (2, \infty)$


**18.** Let $y = |x|$ and then solve as Example 15 (alternate method). **Ans.** $(-3, 3)$

**19.** We have, $\frac{4}{x+1} \leq 3 \leq \frac{6}{x+1}$

$\Rightarrow \quad 4 \leq 3(x+1) \leq 6$ [$\because x+1 \neq 0 \Rightarrow x \neq -1$]

$\Rightarrow \quad \frac{4}{3} \leq x+1 \leq 2$

$\Rightarrow \quad \frac{4}{3} - 1 \leq x \leq 2 - 1$

$\Rightarrow \quad \frac{1}{3} \leq x \leq 1$

**Ans.** $\left[\frac{1}{3}, 1\right]$

**20.** Solve as Example 3. **Ans.** No solution exists.

**21.** Solve as Example 7. **Ans.** $\left(1, \frac{10}{3}\right)$

**22.** Write the given inequalities as $|x-2| \geq 1$ and $|x-2| \leq 3$ Solve each inequality separately and then take the intersection of their solution sets (and then proceed as Example 13). **Ans.** $[-1, 1] \cup [3, 5]$

**23.** Firstly, put $x+1=0$ and $x=0 \Rightarrow x=-1$ and $x=0$

$\therefore$ $x=-1, 0$ are critical points. So, we will consider three intervals $(-\infty, -1)$, $[-1, 0)$, $[0, \infty)$ and then proceed as Example 14.

**Ans.** $(-\infty, -2) \cup (1, \infty)$

**24.** Solve as Example 15.

**Ans.** $(0, 1] \cup [3, 4)$

**25.** Now, $\frac{x}{2x+1} \geq \frac{1}{4} \Rightarrow \frac{x}{2x+1} - \frac{1}{4} \geq 0$

$\Rightarrow \quad \frac{2x-1}{4(2x+1)} \geq 0$, here $2x+1 \neq 0$, i.e. $x \neq -\frac{1}{2}$

$\Rightarrow \quad (2x-1)(2x+1) \geq 0$

***Case* I** When both are non-negative

$2x-1 \geq 0$ and $2x+1 > 0$

$\Rightarrow \quad x \geq \frac{1}{2}$ and $x > -\frac{1}{2}$

$\Rightarrow \quad x \geq \frac{1}{2}$

$\therefore \quad x \in \left[\frac{1}{2}, \infty\right)$ ...(i)

***Case* II** When both are non-positive

$(2x-1) \leq 0$ and $2x+1 < 0$

$\Rightarrow \quad x \leq \frac{1}{2}$ and $x < \frac{-1}{2}$

$\therefore \quad x \in \left(-\infty, -\frac{1}{2}\right)$ ...(ii)

From Eqs. (i) and (ii), we get

$x \in \left(-\infty, -\frac{1}{2}\right) \cup \left[\frac{1}{2}, \infty\right)$ ...(iii)

Now, $\frac{6x}{4x-1} < \frac{1}{2} \Rightarrow \frac{6x}{4x-1} - \frac{1}{2} < 0$

$\Rightarrow \quad \frac{8x+1}{2(4x-1)} < 0$

$\Rightarrow \quad (8x+1)(4x-1) < 0$

We have, $\quad (8x+1) < 0$ and $(4x-1) > 0$

or $\quad (8x+1) > 0$ and $(4x-1) < 0$

$\Rightarrow \quad \frac{-1}{8} < x < \frac{1}{4}$ ...(iv)

Now, for the solution set of given system of inequalities take common value of $x$ from Eqs. (iii) and (iv), but there is no common value of $x$.

**Ans.** Solution set is a empty set.

# |TOPIC 3|

## Applications of Linear Inequalities in One Variable

In this topic, we shall discuss with various word problems which are based on linear inequalities of one variable. Let us look at the following examples.

### |TYPE I|

### BASED ON MARKS OF STUDENTS

**EXAMPLE |1|** Ram needs a minimum of 360 marks in four tests in a Mathematics course to obtain A grade. In his first three tests, he scored 88, 96 and 79 marks. What should his score be in the fourth test, so that he can make A grade?

***Sol.*** Let Ram scores $x$ marks in the fourth test. Then, the sum of Ram's test scores should be greater than or equal to 360.

i.e. $\quad 88+96+79+x \geq 360$

$\Rightarrow \quad 263+x \geq 360$

$\Rightarrow \quad 263+x-263 \geq 360-263$ [subtracting 263 from both sides]

$\therefore \quad x \geq 97$

Hence, Ram should score 97 or greater than 97 marks in fourth test to obtain A grade.

**EXAMPLE |2|** To receive grade A in a course, one must obtain an average of 90 marks or more in five examinations (each of 100 marks). If Sunita's marks in first four examinations are 87, 92, 94 and 95, then find minimum marks that Sunita must obtain in fifth examinations to get grade A in the course. **[NCERT]**

***Sol.*** Let Sunita obtains $x$ marks in her fifth examination.

Then, $\frac{87+92+94+95+x}{5} \geq 90$

$\Rightarrow \quad 368+x \geq 450$ [multiplying both sides by 5]

$\Rightarrow \quad 368+x-368 \geq 450-368$ [subtracting 368 from both sides]

$\therefore \quad x \geq 82$

Thus, Sunita must obtain a minimum of 82 marks to get grade A in the course.

**EXAMPLE |3|** In the first four examinations, each of 100 marks, Mohan got 94, 73, 72 and 84 marks. If a final average greater than or equal to 80 and less than 90 is needed to obtain a final grade $B$ in a course, then what range of marks in the fifth (last) examination will result, if Mohan receiving grade $B$ in the course?

**Sol.** Let $x$ be the score obtained by Mohan in the last examination.

Then, $80 \le \dfrac{94+73+72+84+x}{5} < 90$

$\Rightarrow \quad 80 \le \dfrac{323+x}{5} < 90$

$\Rightarrow \quad 400 \le 323 + x < 450$ [multiplying both sides by 5]

$\Rightarrow \quad 400 - 323 \le 323 + x - 323 < 450 - 323$ [subtracting 323 from each term]

$\Rightarrow \quad 77 \le x < 127$

Since, the upper limit is 100, therefore the required range is $77 \le x \le 100$.

## |TYPE II|

## BASED ON NUMBERS

**EXAMPLE |4|** The sum of three consecutive integers must not be more than 12. What are the integers?

**Sol.** Let the three consecutive integers be $x, (x+1)$ and $(x+2)$. Since, sum of these integers must not be more than 12, i.e. $x+(x+1)+(x+2)$ cannot exceed 12, therefore, we have

$\Rightarrow \quad x+(x+1)+(x+2) \le 12$

$\Rightarrow \quad 3x + 3 \le 12 \Rightarrow 3x + 3 - 3 \le 12 - 3$ [subtracting 3 from both sides]

$\Rightarrow \quad 3x \le 9$

$\therefore \quad x \le 3$ [dividing both sides by 3]

**EXAMPLE |5|** Find all pairs of consecutive even positive integers, both of which are larger than 5, such that their sum is less than 23.

**Sol.** Let $x$ be the smaller of the two positive even integers, so that the other one is $x + 2$, then we should have

$x > 5$ ...(i)

$x + 2 > 5$ ...(ii)

and their sum, $x + (x+2) < 23$ ...(iii)

From inequalities (i) and (ii), we get

$x > 5$ ...(iv)

From inequality (iii), we get $2x + 2 < 23$

$\Rightarrow \quad 2x + 2 - 2 < 23 - 2$ [subtracting 2 from both sides]

$\Rightarrow \quad 2x < 21$

$\Rightarrow \quad x < 10.5$ [dividing both sides by 2]...(v)

Now, from inequalities (iv) and (v), we get $5 < x < 10.5$

Since, $x$ is an even number. So $x$ can take the values 6, 8 and 10.

Hence, the required possible pairs will be (6, 8), (8, 10) and (10, 12).

**EXAMPLE |6|** Find all the pairs of consecutive odd positive integers, both of which are smaller than 10, such that their sum is more than 11. **[NCERT]**

**Sol.** Let $x$ be the smaller of two positive odd integers, so that other one is $x + 2$.

We should have $\quad x < 10$ ...(i)

$x + 2 < 10 \Rightarrow x < 8$ ...(ii)

and $\quad x + (x+2) > 11$ ...(iii)

From inequalities (i) and (ii), we get

$x < 8$ ...(iv)

From inequality (iii), we get

$2x + 2 > 11$

$\Rightarrow \quad 2x > 9 \Rightarrow x > 4.5$ ...(v)

From inequalities (iv) and (v), we get

$4.5 < x < 8$

Since, $x$ is an odd number, so $x$ can take the values 5 and 7.

Hence the required possible pairs will be (5, 7) and (7, 9).

## |TYPE III|

## BASED ON PROFIT, COST AND REVENUE

**EXAMPLE |7|** The cost and revenue functions of a product are given by $C(x) = 20x + 4000$ and $R(x) = 60x + 2000$ respectively, where $x$ is the number of items produced and sold. How many items must be sold to realise some profit? **[NCERT Exemplar]**

**Sol.** We know that, Profit = Revenue − Cost

$\therefore$ In order to realise some profit, revenue should be greater than the cost.

Thus, we should have $R(x) > C(x)$

$\Rightarrow \quad 60x + 2000 > 20x + 4000$

$\Rightarrow \quad 60x + 2000 - 20x > 20x + 4000 - 20x$ [subtracting $20x$ from both sides]

$\Rightarrow \quad 40x + 2000 > 4000$

$\Rightarrow \quad 40x + 2000 - 2000 > 4000 - 2000$ [subtracting 2000 from both sides]

$\Rightarrow \quad 40x > 2000$

$\Rightarrow \quad \dfrac{40x}{40} > \dfrac{2000}{40}$ [dividing both sides by 40]

$\Rightarrow \quad x > 50$

Hence, the manufacturer must sell more than 50 items to realise some profit.

## |TYPE IV|

### BASED ON TEMPERATURE

**EXAMPLE |8|** A solution is to be kept between 68°F and 77°F. What is the range of temperature in degree Celsius (C), if the Celsius/Fahrenheit (F) conversion formula is given by $F = \frac{9}{5}C + 32$? **[NCERT]**

**Sol.** It is given that, $68 < F < 77$

On putting $F = \frac{9}{5}C + 32$, we get

$$68 < \frac{9}{5}C + 32 < 77$$

$$\Rightarrow \quad 68 - 32 < \frac{9}{5}C < 77 - 32$$

[subtracting 32 from each term]

$$\Rightarrow \quad 36 < \frac{9}{5}C < 45$$

$$\Rightarrow \quad 36 \times \frac{5}{9} < C < 45 \times \frac{5}{9}$$

$\left[\text{multiplying } \frac{5}{9} \text{ in each term}\right]$

$$\Rightarrow \quad 4 \times 5 < C < 5 \times 5$$

$$\Rightarrow \quad 20 < C < 25$$

Hence, the required range of temperature is between 20°C and 25°C.

**EXAMPLE |9|** A solution is to be kept between 40°C and 45°C. What is the range of temperature in degree Fahrenheit, if the conversion formula is

$$F = \frac{9}{5}C + 32?$$

**Sol.** We have, $40 < C < 45$

On multiplying each term by $\frac{9}{5}$, we get

$$40 \times \frac{9}{5} < \frac{9}{5}C < \frac{9}{5} \times 45$$

$$\Rightarrow \quad 8 \times 9 < \frac{9}{5}C < 9 \times 9$$

$$\Rightarrow \quad 72 < \frac{9}{5}C < 81$$

On adding 32 in each term, we get

$$72 + 32 < \frac{9}{5}C + 32 < 81 + 32$$

$$\Rightarrow \quad 104 < F < 113 \qquad \left[\because F = \frac{9}{5}C + 32\right]$$

Hence, the required range of temperature is between 104°F and 113°F.

## |TYPE V|

### BASED ON GEOMETRY

**EXAMPLE |10|** The longest side of a triangle is twice the shortest side and the third side is 2 cm longer than the shortest side. If the perimeter of the triangle is more than 166 cm then find the minimum length of the shortest side. **[NCERT Exemplar]**

**Sol.** Let the length of shortest side be $x$ cm.

Then, according to question, we have

Length of longest side = $2x$ cm

and length of third side = $(x + 2)$ cm

Since, the perimeter of the triangle is more than 166 cm.

$$\therefore \quad 2x + x + (x + 2) > 166$$

$$\Rightarrow \quad 4x + 2 > 166$$

$$\Rightarrow \quad 4x > 164 \quad \text{[subtracting 2 from both sides]}$$

$$\Rightarrow \quad x > 41 \quad \text{[dividing both sides by 4]}$$

Hence, the length of the shortest side should be greater than 41 cm.

**EXAMPLE |11|** A man wants to cut three lengths from a single piece of board of length 91 cm, the second length is to be 3 cm longer than the shortest and third length is to be twice as long as the shortest. What are the possible lengths for the shortest board, if third piece is to be atleast 5 cm longer than the second? **[NCERT]**

**Sol.** Let the length of the shortest piece be $x$ cm, so that the lengths of second and third piece are $(x + 3)$ cm and $2x$ cm, respectively.

Then, $\quad x + (x + 3) + 2x \le 91 \quad$ ...(i)

and $\quad 2x \ge (x + 3) + 5 \quad$ ...(ii)

From inequality (i), we get

$$4x + 3 \le 91$$

$$\Rightarrow \quad 4x + 3 - 3 \le 91 - 3$$

[subtracting 3 from both sides]

$$\Rightarrow \quad 4x \le 88$$

[dividing both sides by 4]

$$\Rightarrow \quad \frac{4x}{4} \le \frac{88}{4}$$

$$\Rightarrow \quad x \le 22 \quad \text{...(iii)}$$

From inequality (ii), we get

$$2x \ge x + 8$$

$$\Rightarrow \quad 2x - x \ge x + 8 - x$$

[subtracting $x$ from both sides]

$$\Rightarrow \quad x \ge 8 \quad \text{...(iv)}$$

From inequalities (iii) and (iv), we get

$$8 \le x \le 22$$

Hence, the shortest piece must be atleast 8 cm long but not more than 22 cm long.

## | TYPE VI |

### BASED ON DILUTION OF ACID

**EXAMPLE |12|** A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 litres of the 8% solution, how many litres of the 2% solution will have to be added? **[NCERT]**

**Sol.** Let $x$ be the number of litres of 2% boric acid solution.

Then, total mixture $= (640 + x)$ litres

According to the question,

$$2\% \text{ of } x + 8\% \text{ of } 640 > 4\% \text{ of } (640 + x)$$

$\Rightarrow \quad \frac{2x}{100} + \frac{8}{100} \times 640 > \frac{4}{100}(640 + x)$

$\Rightarrow \quad 2x + 5120 > 2560 + 4x$ [multiplying both sides by 100]

$\Rightarrow \quad 2x + 5120 - 2x > 2560 + 4x - 2x$ [subtracting $2x$ from both sides]

$\Rightarrow \quad 5120 > 2560 + 2x$

$\Rightarrow \quad 5120 - 2560 > 2560 + 2x - 2560$ [subtracting 2560 from both sides]

$\Rightarrow \quad 2560 > 2x$

$\Rightarrow \quad 2x < 2560$

$\Rightarrow \quad x < 1280$ [dividing both sides by 2] ...(i)

Also, $2\%$ of $x + 8\%$ of $640 < 6\%$ of $(640 + x)$

$\Rightarrow \quad \frac{2}{100} \times x + \frac{8}{100} \times 640 < \frac{6}{100} \times (640 + x)$

$\Rightarrow \quad 2x + 5120 < 3840 + 6x$ [multiplying both sides by 100]

$\Rightarrow \quad 2x + 5120 - 2x < 3840 + 6x - 2x$ [subtracting $2x$ from both sides]

$\Rightarrow \quad 5120 < 3840 + 4x$

$\Rightarrow \quad 5120 - 3840 < 3840 + 4x - 3840$ [subtracting 3840 from both sides]

$\Rightarrow \quad 1280 < 4x$ or $4x > 1280$

$\Rightarrow \quad \frac{4x}{4} > \frac{1280}{4}$ [dividing both sides by 4]

$\Rightarrow \quad x > 320$ ...(ii)

Now, from inequalities (i) and (ii), we get

$$320 < x < 1280$$

Thus, the number of litres of the 2% boric acid solution will have to be greater than 320 litres and less than 1280 litres.

**EXAMPLE |13|** How many litres of water will have to be added to 1125 L of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content? **[NCERT]**

**Sol.** Let $x$ L of water is required to be added.

Then, total mixture $= (x + 1125)$ L.

Clearly, the amount of acid in the resulting mixture is 45% of 1125 L. Since, the resulting mixture contain more than 25% but less than 30% acid content. Therefore, we have

$$45\% \text{ of } 1125 > 25\% \text{ of } (x + 1125) \quad \text{...(i)}$$

and $\quad 45\% \text{ of } 1125 < 30\% \text{ of } (x + 1125)$ ...(ii)

From Eq. (i), we have

$$\frac{45}{100} \times 1125 > \frac{25}{100} \times (x + 1125)$$

$\Rightarrow \quad 45 \times 1125 > 25 \times (x + 1125)$ [multiplying both sides by 100]

$\Rightarrow \quad 45 \times 1125 > 25x + (25 \times 1125)$

$\Rightarrow \quad 45 \times 1125 - (25 \times 1125) > 25x$ [subtracting $25 \times 1125$ from both sides]

$\Rightarrow \quad 25x < 1125(45 - 25)$

$\Rightarrow \quad 25x < 1125 \times 20$

$\Rightarrow \quad x < \frac{1125 \times 20}{25} = 225 \times 4 = 900$ [dividing both sides by 25]

$\Rightarrow \quad x < 900$ ...(iii)

From Eq. (ii), we have

$$\frac{45}{100} \times 1125 < \frac{30}{100} \times (x + 1125)$$

$$45 \times 1125 < 30x + (30 \times 1125)$$ [multiplying both sides by 100]

$\Rightarrow 45 \times 1125 - (30 \times 1125) < 30x$ [subtracting $30 \times 1125$ from both sides]

$\Rightarrow \quad 30x > 1125(45 - 30)$

$\Rightarrow \quad x > \frac{1125 \times 15}{30} = \frac{1125}{2} = 562.5$

$\Rightarrow \quad x > 562.5$ ...(iv)

From Eqs. (iii) and (iv), we get $562.5 < x < 900$

Thus, the required number of litres of water that is to be added will have to be more than 526.5 L but less than 900 L.

# TOPIC PRACTICE 3

## OBJECTIVE TYPE QUESTIONS

**1** Ravi goes to market with ₹ 200 to buy rice, which is available in packets of 1 kg. The price of one packet of rice is ₹ 30. If $x$ denotes the number of packets of rice which he buys, then the total amount spent by him is ₹ $30x$. The mathematical formulation of the given problem is

(a) $30x > 200$ (b) $30x < 200$
(c) $30x \leq 200$ (d) $30x \geq 200$

**2** The all pairs of consecutive even positive integers, both of which are larger than 5, such that their sum is less than 23, are

(a) (2, 4), (5, 7) and (8, 10)
(b) (3, 5), (6, 8) and (9, 11)
(c) (6, 8), (8, 10) and (10, 12)
(d) None of the above

**3** A company manufactures cassettes. Its cost and revenue functions are $C(x) = 26000 + 30x$ and $R(x) = 43x$, respectively, where $x$ is the number of cassettes produced and sold in a week. How many cassettes must be sold by the company to realise some profit?

(a) more than 2000 (b) less than 2000
(c) more than 5000 (d) less than 5000

**4** In drilling world's deepest hole it was found that the temperature $T$ in degree celcius, $x$ km below the earth's surface was given by $T = 30 + 25(x - 3), 3 \le x \le 15$. At what depth will the temperature be between 155°C and 205°C?

(a) 10 to 12 km (b) 8 to 10 km
(c) 8 to 10 km (d) 15 to 18 km

**5** The length of a rectangle is three times the breadth. If the minimum perimeter of the rectangle is 160 cm, then [NCERT Exemplar]

(a) breadth > 20cm (b) length < 20 cm
(c) breadth ≥ 20 cm (d) length ≤ 20 cm

## SHORT ANSWER Type I Questions

**6** Anu needs a minimum of 450 marks in five tests in statistics course to obtain $B$ grade. In her first four tests, she scored 90, 94, 96 and 88 marks. What should her score be in the 5th test, so that she can make $B$ grade.

**7** The marks obtained by a student of class XI in first and second terminal examinations are 62 and 48, respectively. Find the minimum marks he should get in the annual examination to have an average of atleast 60 marks. [NCERT]

**8** Ravi obtained 70 and 75 marks in first two unit test. Find the minimum marks he should get in the third test to have an average of atleast 60 marks. [NCERT]

**9** In the first four papers each of 100 marks, Rahul got 95, 72, 73 and 83 marks. If he wants an average of greater than 75 marks and less than 80 marks. Find the range of marks he should score in the fifth paper.

**10** A company manufactures cassettes and its cost equation for a week is $C = 300 + 1.5x$ and its revenue equation is $R = 2x$, where $x$ is the number of cassettes sold in a week. How many cassettes must be sold for the company to realise a profit?

**11** IQ (Intelligence Quotient) of a person is given by formula, $IQ = \frac{MA}{CA} \times 100$

where, MA stands for the mental age of the person and CA for the chronological age. If $80 \le IQ \le 140$ for a group of 12 yr children, then find the range of their mental age. [NCERT]

**12** In drilling world's deepest hole it was found that the temperature $T$ in degree celcius $x$ km below the Earth's surface was given by $T(x) = 30 + 25(x - 3)$, where $3 \le x \le 15$. At what depth will the temperature be between 155°C and 205°C?. [NCERT Exemplar]

## SHORT ANSWER Type II Questions

**13** The sum of four consecutive integers must not be more than 22. What are the integers?

**14** Find all the pairs of consecutive positive integers, both of which are larger than 8 such that their sum is less than 28.

**15** Find all pairs of consecutive odd positive integers, both of which are smaller than 18, such that their sum is more than 20.

**16** Find all pairs of consecutive odd natural numbers, both of which are larger than 10, such that their sum is less than 40. [NCERT]

**17** A solution is to be kept between 86°F and 95°F. What is the range of temperature in degree Celsius, if the Celsius (C)/Fahrenheit (F) conversion formula is given by $F = \frac{9}{5}C + 32$.

**18** In an experiment, a solution of hydrochloric acid is to be kept between 30°C and 35°C. What is the range of temperature in degree Fahrenheit, if conversion formula is given by $F = \frac{9}{5}C + 32$, where C and F represent temperature in degree Celsius and degree Fahrenheit, respectively.

## LONG ANSWER Type Question

**20** A solution of 9% acid is to be diluted by adding 3% acid solution to it. The resulting mixture is to be more than 5% but less than 7% acid. If there is 460 litres of the 9% solution. How many litres of 3% solution will have to be added?

**19** The longest side of a triangle is 3 times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the triangle is atleast 61 cm. Find the minimum length of the shortest side. **[NCERT]**

# HINTS & ANSWERS

**1.** (b) Since, Ravi has to buy rice in packets only, he may not be able to spend the entire amount of ₹ 200 because 200 is not a multiple of 30. Hence,

$$30x < 200$$

**2.** (c) Let $x$ be the smaller of the positive even integers, so that the other one is $x + 2$, then we should have

$$x > 5 \quad ...(i)$$

$$x + 2 > 5 \quad ...(ii)$$

and their sum, $x + (x + 2) < 23$ ...(iii)

From inequalities (i) and (ii), we get

$$x > 5 \quad ...(iv)$$

From inequality (iii), we get $2x + 2 < 23$

$\Rightarrow \quad 2x + 2 - 2 < 23 - 2$

[subtracting 2 from both sides]

$\Rightarrow \quad 2x < 21$

$\Rightarrow \quad x < 10.5$ ...(v)

[dividing both sides by 2]

Now, from inequalities (iv) and (v), we get $5 < x < 10.5$.

Since, $x$ is an even number.

So, $x$ can take the values 6, 8 and 10.

Hence, the required possible pairs will be (6, 8), (8, 10) and (10, 12).

**3.** (a) Cost function, $C(x) = 26000 + 30x$

and revenue function, $R(x) = 43x$

For profit, $R(x) > C(x)$

$\Rightarrow \quad 26000 + 30x < 43x$

$\Rightarrow \quad x > \frac{26000}{13}$

$\therefore \quad x > 2000$

Hence, more than 2000 cassettes must be produced to get profit.

**4.** (b) Given that, $T = 30 + 25(x - 3), 3 \le x \le 15$

According to the question, $155 < T < 205$

$\Rightarrow \quad 155 < 30 + 25(x - 3) < 205$

$\Rightarrow \quad 8 < x < 10$

Hence, at the depth 8 to10 km temperature lies between 155° to 205°C.

**5.** (c) Let breadth of rectangle be $x$ cm.

$\therefore$ Length of rectangle $= 3x$

Perimeter of rectangle $= 2$(Length + Breadth)

$= 2(x + 3x) = 8x$

Given, Perimeter $\ge 160$ cm

$$8x \ge 160$$

Dividing both sides by 8,

$$x \ge 20 \text{ cm}$$

**6.** Solve as Example 1. **Ans.** 82 marks

**7.** Let $x$ marks be obtained by students in the annual examination. Then, $\frac{62 + 48 + x}{3} \ge 60$ and then proceed as Example 2. **Ans.** 70

**8.** Solve as Example 1. **Ans.** 35 marks

**9.** Let Rahul scores $x$ marks in fifth paper.

Then, $75 < \frac{95 + 72 + 73 + 83 + x}{5} < 80$

Now, proceed as Example 3.

**Ans.** Rahul must score between 52 and 77 marks.

**10.** Profit = Revenue − Cost

and in order to realise some profit, revenue should be greater than the cost. Now, proceed as Example 7.

**Ans.** More than 600 cassettes.

**11.** Also, $IQ = \frac{MA}{CA} \times 100$

$\therefore \quad 80 \le \frac{MA}{CA} \times 100 \le 140 \Rightarrow 80 \le \frac{MA}{12} \times 100 \le 140$

**Ans.** $9.6 \le MA \le 16.8$

**12.** Let at s km below the Earth's surface, the temperature is between 155°C and 205°C.

$\therefore \quad 155 < T(s) < 205$

$\Rightarrow 155 < 30 + 25(s - 3) < 205$

**Ans.** $8 < s < 10$.

**13.** Solve as Example 4. **Ans.** Less than or equal to 4.

**14.** Solve as Example 5. **Ans.** (10, 12), (12, 14)

**15.** Solve as Example 6.

**Ans.** (11, 13), (13, 15) and (15, 17)

**16.** Solve as Example 6.

**Ans.** (11, 13), (13, 15), (15, 17), (17, 19)

**17.** Solve as Example 8.

**Ans.** Between 30°C and 35°C

**18.** Solve as Example 9

**Ans.** Between 59°F and 77°F

**19.** Let the length of shortest side be $x$ cm. Then, we have $3x + x + (3x - 2) \ge 61$ and then proceed as Example 10.

**Ans** 9 cm.

**20.** Solve as Example 12.

**Ans.** More than 230 L but less than 920 L.

## SUMMARY

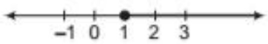
- Two real numbers or two algebraic expressions related by the symbols >, <, ≥ or ≤ form an inequality.
- Types of Inequalities
  (i) An inequality which does not involve any variable is called a numerical inequality.
  (ii) An inequality which have variables is called literal inequality.
  (iii) An inequality which have only < or > is called strict inequality.
  (iv) An inequality which have only ≥ or ≤ is called slack inequality.
- An inequality is said to be **linear inequality** if each variable occurs in first degree only and there is no term involving the product of the variables.
- A linear inequality which has only one (or two) variable, is called **linear inequality in one (or two) variable(s).**
- **Properties of Numerical Inequalities**
  (i) Equal numbers may be **added** or **subtracted** from both sides of an inequality.
  (ii) If both sides of an inequality are **multiplied** or **divided** by same positive number, then the sign of inequality remains the same. But when both sides are multiplied or divided by a negative number, then the sign of inequality is reversed.
- Any **solution of an inequality in one (or two) variable(s)** is a value of variable which makes it a true statement.
- The set of all solutions of an inequality, is the **solution set** of the inequality.
- The **solution set of a system of linear inequalities** in one variable is defined as the intersection of the solution set of the linear inequalities in the system.
- To represent $x < a$ (or $x > a$) on a number line, put a circle on the number $a$ and dark line to the left (or right) of the number $a$.
- To represent $x \leq a$ (or $x \geq a$) on a number line, put a dark circle on the number $a$ and dark line to the left (or right) of the number $x$.

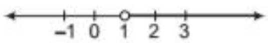
# CHAPTER PRACTICE

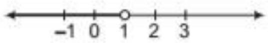
## OBJECTIVE TYPE QUESTIONS

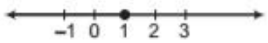
**1.** Two real numbers or two algebraic expressions related by the symbol '<', '>', '≤' or '≥' forms an
(a) equation
(b) inequality
(c) set
(d) None of the above

**2.** The graph of the solutions of inequality $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$ on number line is

(a)



(b)



(c)



(d)



**3.** The solution set of $\frac{2x-1}{3} \geq \left(\frac{3x-2}{4}\right) - \left(\frac{2-x}{5}\right)$ is
(a) $(-\infty, 2)$ (b) $(-\infty, 2]$
(c) $\left(-\infty, -\frac{1}{2}\right)$ (d) $\left(-\infty, -\frac{1}{2}\right]$

**4.** The set of values of $x$ satisfying $2 \leq |x-3| < 4$ is
(a) $(-1, 1] \cup [5, 7)$
(b) $-4 \leq x \leq 2$
(c) $-1 < x < 7$ or $x \geq 5$
(d) $x < 7$ or $x \geq 5$
(e) $-\infty < x \leq 1$ or $5 \leq x < \infty$

**5.** The solution set of the linear inequalities $-15 < \frac{3(x-2)}{5} \leq 0$, is
(a) $(-23, 2]$
(b) $(-20, 4)$
(c) $[-23, 2)$
(d) None of the above

## SHORT ANSWER Type I Questions

**6.** Solve the inequality $3(2-x) \geq 2(1-x)$ and show the graph of solution on number line. **[NCERT]**

**7.** Solve $12 + 1\frac{5}{6}x \leq 5 + 3x$, when (i) $x \in N$ (ii) $x \in R$.

**8.** Solve the inequality $\frac{2x-3}{4} + 9 \geq 3 + \frac{4x}{3}$ and show the graph of the solution on number line.

**9.** Solve for $x: 4x + 3 \geq 2x + 17, 3x - 5 < -2$.

**10.** Solve the following system of linear inequalities. $\frac{4x}{3} - \frac{9}{4} < x + \frac{3}{4}$ and $\frac{7x-1}{3} - \frac{7x+2}{6} > x$

**11.** Solve for $x: -5 \leq \frac{2-3x}{4} \leq 9$. **[NCERT Exemplar]**

**12.** Solve $2x - 1 > x + \frac{7-x}{3} > 2, x \in R$.

**13.** Anshu obtained 73, 67 and 72 marks in the Mathematics test. How many marks should he get in his fourth test, so as to have an average of at least 75?

## SHORT ANSWER Type II Questions

**14.** Solve for $x: \frac{x-3}{x-5} > 0$.

**15.** Solve $\frac{5}{x-2} > 3$ and represent the solution set on number line.

**16.** Solve the following system of inequalities.
$$\frac{2x+1}{7x-1} > 5, \frac{x+7}{x-8} > 2$$

**17.** Solve $|x| < 4$ and represent the solution set on the number line.

**18.** Solve the inequality $\left|\frac{2}{x-4}\right| > 1; x \neq 4$.

**19.** Solve $\frac{|x-1|}{x+2} < 1$.

**20.** A company manufactures cassettes. Its cost and revenue functions are $C(x) = 26000 + 30x$ and $R(x) = 43x$, respectively, where $x$ is the number of cassettes produced and sold in a week. How many cassettes must be sold by the company to realise some profit? **[NCERT Exemplar]**

**21.** The water acidity in a pool is considered normal when the average pH reading of three daily measurements is between 8.2 and 8.5. If the two pH readings are 8.48 and 8.35, find the range of pH value for the third reading that will result in the acidity level being normal. **[NCERT Exemplar]**

**22.** A manufacturer has 600 L of a 12% solution of acid. How many litres of a 30% acid solution must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18%? **[NCERT]**

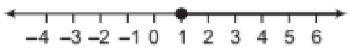
# HINTS & ANSWERS

**1.** (b) By definition of inequality.

**2.** (a) We have, $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$

or $\quad 5x \geq 5$ or $x \geq 1$

The graphical representation of solutions is given in figure.




**3.** (b) We have, $\frac{2x-1}{3} \geq \left(\frac{3x-2}{4}\right) - \left(\frac{2-x}{5}\right)$

Taking LCM in RHS,

$$\frac{2x-1}{3} \geq \frac{5(3x-2)-4(2-x)}{20}$$

$\Rightarrow \quad 40x - 20 \geq 57x - 54$

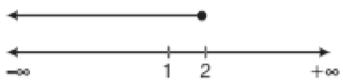
$\quad 40x - 57x \geq -54 + 20$

$\Rightarrow \quad -17x \geq -34$

Dividing both sides by $-17$,

$\Rightarrow \quad x \leq \frac{-34}{-17}$

$\Rightarrow \quad x \leq 2$




$\therefore$ Solution set is $(-\infty, 2]$.

**4.** (a) We have, $2 \leq |x-3| < 4$

*Case* **I** If $x < 3$, then

$2 \leq |x-3| < 4$

$-1 < x \leq 1$

$\Rightarrow \quad x \in (-1, 1]$

*Case* **II** If $x > 3$, then

$2 \leq |x-3| < 4$

$\Rightarrow \quad 2 \leq x - 3 < 4$

Adding 3 on both sides,

$\Rightarrow \quad 5 \leq x < 7$

Hence, the solution set of given inequality is $x \in (-1, 1] \cup [5, 7)$.

**5.** (a) We have, $-15 < \frac{3(x-2)}{5} \leq 0$

$\Rightarrow \quad -15 \times 5 < \frac{3(x-2)}{5} \times 5 \leq 0 \times 5$

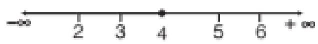
$\Rightarrow \quad -23 < x \leq 2$

$\therefore \quad x \in (-23, 2]$

Hence, the solution set is $(-23, 2]$.

**6.** $3(2-x) \geq 2(1-x) \Rightarrow 6 - 2 \geq 3x - 2x \Rightarrow x \leq 4$

**Ans.** $(-\infty, 4]$




**7.** $12 + \frac{11x}{6} \leq 5 + 3x \Rightarrow \frac{7x}{6} \geq 7 \Rightarrow x \geq 6$

**Ans.** (i) $\{6, 7, 8, 9, \ldots\}$ (ii) $[6, \infty)$

**8.** Solve as Q. 6. **Ans.** $(-\infty, 9.9]$


**9.** $4x + 3 \geq 2x + 17$ and $3x - 5 < -2$

$\Rightarrow 2x \geq 14$ and $3x < 3 \Rightarrow x \geq 7$ and $x < 1$

**Ans.** No solution exists.

**10.** Solve as Example 2 of Topic 2. **Ans.** (4, 9)

**11.** $-5 \leq \frac{2-3x}{4} \leq 9$

$\Rightarrow \quad -20 \leq 2 - 3x \leq 36$

$\Rightarrow \quad -22 \leq -3x \leq 34$

$\Rightarrow \quad -34 \leq 3x \leq 22$

**Ans.** $\left[\frac{-34}{3}, \frac{22}{3}\right]$

**12.** $2x - 1 > \frac{2x+7}{3} > 2 \Rightarrow 6x - 3 > 2x + 7 < 6$

$\Rightarrow \quad 6x - 3 > 2x + 7$ and $2x + 7 > 6$

$\Rightarrow \quad 4x > 10$ and $2x > -1$

$\Rightarrow \quad x > \frac{5}{2}$ and $x > -\frac{1}{2}$

**Ans.** $\left]\frac{5}{2}, \infty\right[$

13. $\dfrac{73+67+72+x}{4} \geq 75$ **Ans.** 88 marks

14. $(x-3)(x-5)>0$

*Case* I When both are positive

$\therefore\ x>3$ and $x>5 \Rightarrow x>5$

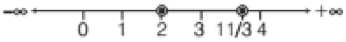
*Case* II When both are negative

$x-3<0$ and $x-5<0 \Rightarrow x<3$ and $x<5 \Rightarrow x<3$

**Ans.** $(-\infty, 3) \cup (5, \infty)$

15. $\dfrac{5}{x-2}-3>0 \Rightarrow \left(\dfrac{11-3x}{x-2}\right)>0$

$\Rightarrow \quad 2<x<\dfrac{11}{3}$ **Ans.** $\left]2, \dfrac{11}{3}\right[$




16. $\dfrac{2x+1}{7x-1}-5>0,\ \dfrac{x+7}{x-8}-2>0$

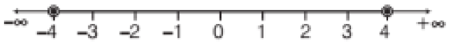
$\Rightarrow \quad \dfrac{-33x+6}{7x-1}>0,\ \dfrac{-x+23}{(x-8)}>0$

$\Rightarrow (-33x+6)(7x-1)>0,\ (-x+23)(x-8)>0$

$\Rightarrow \quad x>7,\ x<23$

**Ans.** No solution exists

17. $|x|<4 \Rightarrow -4<x<4$ **Ans.** $]-4, 4[$




18. $\left|\dfrac{2}{x-4}\right|>1 \Rightarrow \dfrac{2}{x-4}<-1,\ \dfrac{2}{x-4}>1$

$\Rightarrow \quad \dfrac{2}{x-4}+1<0,\ \dfrac{2}{x-4}-1>0$

$\Rightarrow \quad \dfrac{-2+x}{x-4}<0,\ \dfrac{6-x}{x-4}>0$

$\Rightarrow \ (-2+x)(x-4)<0$ and $(6-x)(x-4)>0$

Further, solve as Q. 15. **Ans.** $(2, 4) \cup (4, 6)$

19. Solve as Example 16 of Topic 1.

**Ans.** $(-\infty, -2) \cup \left(\dfrac{-1}{2}, \infty\right)$

20. For profit, $R(x)>C(x)$ **Ans.** More than 2000.

21. $8.2<\dfrac{8.48+8.35+x}{3}<8.5$ **Ans.** Between 7.77 and 8.77.

22. Greater than 120 L and less than 300 L.